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On quantum corrections to Chern-Simons spinor electrodynamics

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Abstract. We make a detailed investigation on the quantum corrections to Chern-Simons spinor electrodynamics. Starting from Chern-Simons spinor quantum electrodynamics with the Maxwell term $-1/(4\gamma)\int d^3x F_{\mu\nu}F^{\mu\nu}$ and by calculating the vacuum polarization tensor, electron self-energy and on-shell vertex, we explicitly show that the Ward identity is satisfied and hence verify that the physical quantities are independent of the procedure of taking $\gamma\to\infty$ at tree and one-loop levels. In particular, we find the three-dimensional analogue of the Schwinger anomalous magnetic moment term of the electron produced from the quantum corrections.

I Introduction

There is a relatively long history for (Abelian or Non-Abelian) Chern-Simons (CS) theory and its relevant theories to become popular in physics. At early stage they appeared as the high-temperature limit of four dimensional field models, where Maxwell-Chern-Simons theory can be regarded as an effective theory of QCD and the electroweak model [1]. Further its more striking aspect had been found: in three-dimensional space-time the CS term can provide a topological mass for the gauge field in a gauge invariant way as an alternative to Higgs mechanism [2,3]. In recent years the revival to the study of CS theory, on one hand, is due to Witten's work [4] in which a connection between CS theory and 2-dimensional conformal invariant field theory was found; on the other hand, owing to the non-invariance of CS term under P and T transformations and especially its topological character, it can be used to describe the dynamics of anyon particles so that it has been favoured by physicists to solve some problems in condensed matter theory such as the fractional quantum Hall effect and the high temperature superconductivity [5]. It is also proved that CS term coupled to scalar matter is useful in the field-theoretic formulation of the Aharonov-Bohm effect [6] and the three-dimensional analogue of Coleman-Weinberg mechanism is explored up to two-loop [7].

In this paper we shall present a detailed investigation on the one-loop quantum correction of one-loop CS spinor electrodynamics. We start from the action with Maxwell term

$$S = \frac{\mu}{2}\int d^3x\,\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda - \frac{1}{4\gamma}\int d^3x\,F_{\mu\nu}F^{\mu\nu} + \int d^3x\left[\bar{\psi}(i\hat{\partial}+e\hat{A}-m)\psi\right] - \frac{1}{2\alpha}\int d^3x(\partial_\mu A^\mu)^2, \tag{1}$$

where (and in what follows) $\hat{A}\equiv\gamma^\alpha A_\alpha$, μ is the statistical parameter and we choose the Lorentz gauge condition $\partial_\mu A^\mu = 0$. The notation is the same as that in [3],

$$\gamma_\mu = i\sigma_\mu,\ \gamma_\mu\gamma_\nu = g_{\mu\nu} - i\epsilon_{\mu\nu\rho}\gamma^\rho\ ,\quad g_{\mu\nu} = \mathrm{diag}(1,-1,-1). \tag{2}$$

It should be stressed that the introduction of Maxwell term plays a two-fold role: on one hand, it provides a mathematically correct path-integral quantization of the CS theory in Euclidean region, since the pure CS term contains the non-positive definite first order differential operator; on the other hand, as a higher order derivative term, it provides a gauge invariant regularization. However, this regularization is not enough to make one-loop amplitude finite, another regularization must be implemented. Here we shall adopt dimensional regularization.

The model (1) has been studied by many authors [8], especially the case where CS term is absent (i.e. pure QED_3). However they mainly consider the dynamical mass generation, the chiral symmetry and parity breaking by quantum corrections. A complete investigation on its quantum correction still lacks such as the explicit verification of Ward identity and whether there exists the three-dimensional analogue of Schwinger's anomalous magnetic moment term, all of which depend on an explicit analytical calculation of the vertex correction. To our knowledge, up to now there appears no analytic result on this part. We are further motivated by the result of pure non-Abelian

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CS theory, where a different order in taking $\gamma\to\infty$ can result in a different finite renormalization of the statistical parameter [9]. It is desirable to see whether this case happens in CS spinor electrodynamics too. As we know, in quantum electrodynamics, the Ward identity means

$$Z_1 = Z_2, \tag{3}$$

where Z_2 and Z_1 are the electron wave function renormalization constant and vertex renormalization constant respectively. So if the Ward identity is satisfied, the renormalization of coupling constant is only relevant to the gauge field wave function renormalization constant Z_3:

$$e_R = \sqrt{Z_3} Z_1 Z_2^{-1} e = \sqrt{Z_3} e. \tag{4}$$

Since Z_3 is independent of the introduction of Maxwell term, so the Ward identity means that the physical quantities have nothing to do with the order of taking $\gamma\to\infty$. In particular it is very interesting to see whether there exists an anomalous magnetic moment term, since it can produce a new interaction between anyons that will lead to unusual planar dynamics [10,11]. This may be helpful to understand the mechanisms of fractional quantum Hall effects and high temperature superconductivity.

The Feynman rules are listed as follows

– gauge field propagator

$$\tilde{D}^{(0)}_{\mu\nu}(p) = -i\frac{\gamma}{p^2-\mu^2\gamma^2}\left[i\mu\gamma\epsilon_{\mu\nu\rho}\frac{p^\rho}{p^2} + g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right], \tag{5}$$

where we choose Landau gauge ($\alpha = 0$) to avoid infrared singularity [3,12]. In the limit of $\gamma\to\infty$, we have

$$D^{(0)}_{\mu\nu}(p) = -\frac{1}{\mu}\epsilon_{\mu\nu\rho}\frac{p^\rho}{p^2}. \tag{6}$$

– electron propagator

$$S^{(0)}(p) = i\frac{\hat{p}+m}{p^2-m^2}. \tag{7}$$

– the vertex

$$-ie\gamma_\mu(2\pi)^3\delta^{(3)}(p+q+r). \tag{8}$$

In Sect. II, starting from the classical action (1), we calculate the vacuum polarization tensor and electron self-energy correction and define the finite renormalization constants relevant to them. In Sect. III, we compare our results obtained in dimensional regularization with those obtained in Pauli-Villars regularization and expressed in spectral representation and find the results are identical, this shows the gauge invariant regularization scheme independence. Section IV is devoted to a detail calculation of mass-shell vertex correction. It is explicitly shown that the Ward identity is satisfied on mass-shell. Especially, we find the three dimensional analogue of anomalous magnetic moment term. In Sect. V we turn to pure CS spinor electrodynamics (i.e. taking $\gamma\to\infty$ at tree level) and we verify that the Ward identity is still satisfied, which shows that the physical quantities are independent of the order of taking large-γ limit. Section VI contains the conclusions and some discussions on higher order results.

II One-loop vacuum polarization and self-energy

A Polarization tensor

The polarization tensor gets contribution from the electron loop and its amplitude is

$$\begin{aligned} &i\Pi_{\mu\nu}(p) \\ &= -e^2\int\frac{d^nq}{(2\pi)^n}\frac{\mathrm{Tr}\gamma_\nu[(\hat{q}+\hat{p})+m]\gamma_\mu(\hat{q}+m)}{[(q+p)^2-m^2][q^2-m^2]} \\ &= -2e^2\int\frac{d^nq}{(2\pi)^n} \\ &\frac{-im\epsilon_{\mu\nu\rho}p^\rho + 2q_\mu q_\nu + q_\mu p_\nu + q_\nu p_\mu + [m^2 - q\cdot(q+p)]g_{\mu\nu}}{(q^2-m^2)[(q+p)^2-m^2]}. \end{aligned} \tag{9}$$

The standard calculation gives

$$\begin{aligned} &\Pi_{\mu\nu}(p) \\ &= i\epsilon_{\mu\nu\rho}p^\rho\Pi_{\mathrm{o}}(p^2) + (p^2g_{\mu\nu} - p_\mu p_\nu)\Pi_{\mathrm{e}}(p^2) \\ &= \frac{e^2}{4\pi}\left\{i\epsilon_{\mu\nu\rho}p^\rho\frac{m}{p}\ln\frac{1+p/(2m)}{1-p/(2m)}\right. \\ &\quad -(p^2g_{\mu\nu}-p_\mu p_\nu)\left[-\frac{m}{p^2}+\left(\frac{1}{4p}+\frac{m^2}{p^3}\right)\right. \\ &\quad \left.\left.\times\ln\frac{1+p/(2m)}{1-p/(2m)}\right]\right\}. \end{aligned} \tag{10}$$

B Electron self-energy

The Feynman integral for electron self-energy is read as follows

$$\begin{aligned} &-i\tilde{\Sigma}(p,m,\gamma) \\ &= -e^2\gamma\int\frac{d^nq}{(2\pi)^n} \\ &\quad\times\frac{\gamma_\nu[(\hat{q}+\hat{p})+m]\gamma_\mu[i\mu\gamma\epsilon^{\mu\nu\rho}q_\rho + q^2g^{\mu\nu} - q^\mu q^\nu]}{[(q+p)^2-m^2]q^2(q^2-\mu^2\gamma^2)} \\ &= -e^2\gamma\int\frac{d^nq}{(2\pi)^n} \\ &\quad\times\frac{-\hat{q}[m\mu\gamma + q\cdot(q+p)] + q^2m + \mu\gamma q\cdot(q+p)}{[(q+p)^2-m^2]q^2(q^2-\mu^2\gamma^2)}. \end{aligned} \tag{11}$$

Using the identities

$$\begin{aligned} \frac{1}{q^2(q^2-\mu^2\gamma^2)} &= \frac{1}{\mu^2\gamma^2}\left(\frac{1}{q^2-\mu^2\gamma^2} - \frac{1}{q^2}\right), \\ 2q\cdot p &= [(q+p)^2-m^2] - q^2 - (p^2-m^2) \\ &= [(q+p)^2-m^2] - (q^2-\mu^2\gamma^2) \\ &\quad -(p^2-m^2+\mu^2\gamma^2), \end{aligned} \tag{12}$$

Equation (11) can be written as

$$-i\tilde{\Sigma}(p,m,\gamma)$$

$$= -2e^2\gamma\int \frac{d^nq}{(2\pi)^n}\left\{\left(m+\frac{\mu\gamma}{2}-\frac{p^2-m^2}{2\mu\gamma}\right)\right.$$
$$\times\frac{1}{(q^2-\mu^2\gamma^2)[(q+p)^2-m^2]}$$
$$+\frac{1}{2\mu\gamma}\frac{1}{q^2-\mu^2\gamma^2}$$
$$+\frac{p^2-m^2}{2\mu\gamma}\frac{1}{q^2[(q+p)^2-m^2]}$$
$$-\left(\frac{m}{\mu\gamma}+\frac{1}{2}-\frac{p^2-m^2}{2\mu^2\gamma^2}\right)$$
$$\times\frac{\hat{q}}{(q^2-\mu^2\gamma^2)[(q+p)^2-m^2]}$$
$$\left.-\left(\frac{p^2-m^2}{2\mu^2\gamma^2}-\frac{m}{\mu\gamma}\right)\frac{\hat{q}}{q^2[(q+p)^2-m^2]}\right\}. \quad (13)$$

After the integration and the limit of $\gamma\to\infty$, we have

$$\Sigma(p) = \lim_{\gamma\to\infty}\tilde{\Sigma}(p,m,\gamma)$$
$$= \frac{e^2}{4\pi}\left\{2\gamma+\frac{m}{\mu}+\frac{p^2-m^2}{\mu p}\ln\frac{1+p/m}{1-p/m}\right.$$
$$\left.-\frac{\hat{p}}{\mu}\left[\frac{m^2}{p^2}+\frac{m}{p}(1-\frac{m^2}{p^2})\ln\frac{1+p/m}{1-p/m}-\frac{2}{3}\right]\right\}. \quad (14)$$

C Finite renormalization

Now we discuss the finite renormalization of one-loop two point functions. From (6), (10) and the following relation

$$D^{(1)\,-1}_{\mu\nu}(p) = D^{(0)\,-1}_{\mu\nu}(p) - i\Pi_{\mu\nu}(p), \quad (15)$$

we can get the one-loop gauge field propagator

$$D^{(1)}_{\mu\nu}(p) = -i(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2})\frac{\Pi_{\rm e}(p^2)}{\mu^2[1-\Pi_{\rm o}(p^2)]^2-p^2\Pi^2_{\rm e}(p^2)}$$
$$-\epsilon_{\mu\nu\rho}\frac{p^\rho}{p^2}\frac{\mu[1-\Pi_{\rm o}(p^2)]}{\mu^2[1-\Pi_{\rm o}(p^2)]^2-p^2\Pi^2_{\rm e}(p^2)}.$$
$$\approx -i\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)\frac{\Pi_{\rm e}(p^2)}{\mu^2}$$
$$-\epsilon_{\mu\nu\rho}\frac{p^\rho}{p^2\mu}\left(1+\Pi_{\rm o}(p^2)\right). \quad (16)$$

The renormalized propagator should have the following form

$$D^{(1)}_{\mu\nu}(p) \equiv -i(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2})\Pi_1(p^2)$$
$$-\epsilon_{\mu\nu\rho}\frac{p^\rho}{p^2}\left[\frac{Z_3}{\mu_{\rm ph}}+\Pi_2(p^2)\right]. \quad (17)$$

Choosing the renormalization point $p^2 = 0$, we get the finite renormalization of the statistical parameter μ

$$\mu_{\rm ph} = \mu(1+\frac{e^2}{4\pi}), \quad (18)$$

and the one-loop gauge field propgator (up to the order e^2)

$$D^{(1)}_{\mu\nu}(p) = -i\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)\frac{\Pi_{\rm e}(p^2)}{\mu^2_{\rm ph}}$$
$$+\frac{\epsilon_{\mu\nu\rho}p^\rho}{p^2\mu_{\rm ph}}\left(1+\left[\Pi_{\rm o}(p^2)-\Pi_{\rm o}(0)\right]\right), \quad (19)$$

From (17) and (19), one can see

$$\Pi_1(p^2) \equiv \frac{\Pi_{\rm e}(p^2)}{\mu^2_{\rm ph}}$$
$$= \frac{1}{\mu^2_{\rm ph}}\left[\frac{m}{p^2}-\left(\frac{1}{4p}+\frac{m^2}{p^3}\right)\ln\frac{1+p/(2m)}{1-p/(2m)}\right],$$
$$\Pi_2(p^2) \equiv \Pi_{\rm o}(p^2)-\Pi_{\rm o}(0)$$
$$= \frac{e^2}{4\pi}\left(1-\frac{m}{p}\ln\frac{1+p/(2m)}{1-p/(2m)}\right). \quad (20)$$

They can be expressed in terms of the spectral representation [13]:

$$\Pi_1(p^2) = \frac{e^2}{8\pi\mu^2_{\rm ph}}\int_{2m}^{\infty}\frac{da(1+4m^2/a^2)}{a^2-p^2-i\epsilon},$$
$$\Pi_2(p^2) = \frac{e^2mp^2}{2\pi\mu_{\rm ph}}\int_{2m}^{\infty}\frac{da}{a^2(p^2-a^2+i\epsilon)}. \quad (21)$$

Correspondingly, we can find from (17) and (19),

$$Z_3 = 1. \quad (22)$$

We observe that the terms $\sim \Pi_e(p^2)$ appear in (19); this means the quantum correction generates the parity-even part of the gauge field propagator.

As for the finite renormalization of electron self-energy, it is defined by the usual mass-shell renormalization condition

$$\Sigma_R(p)|_{\hat{p}=m_{\rm ph}} = 0, \ \frac{\partial}{\partial\hat{p}}\Sigma_R(p)|_{\hat{p}=m_{\rm ph}} = 0. \quad (23)$$

Thus the self-energy can be written as the expansion around $\hat{p} = m_{\rm ph}$,

$$\Sigma(p) = \delta m - (Z_2^{-1}-1)(\hat{p}-m_{\rm ph}) + Z_2^{-1}\Sigma_R(p) \quad (24)$$

and the one-loop electron propagator is

$$S^{(1)}(p) = i\frac{Z_2}{\hat{p}-m_{\rm ph}-\Sigma_R(p)}$$
$$= i\left[\frac{Z_2}{\hat{p}-m_{\rm ph}}+\tilde{\Sigma}_R(p)\right]. \quad (25)$$

From the one-loop correction (14), the physical mass, electron wave function renormalization constant and the radiative correction are (up to the order e^2) given by

$$m_{\rm ph} = m-\delta m = m-\frac{e^2}{2\pi}(\gamma+\frac{m}{3\mu}),$$

$$Z_2=1+\frac{e^2}{4\pi}\frac{5}{3\mu},$$

$$\Sigma_R(p)=\frac{e^2}{4\pi}\left\{\frac{2m_{\rm ph}}{\mu}+\frac{p^2-m^2_{\rm ph}}{\mu p}\ln\frac{1+p/m_{\rm ph}}{1-p/m_{\rm ph}}\right.$$
$$-\frac{\hat{p}}{\mu}\left[1+\frac{m^2_{\rm ph}}{p^2}+\frac{m_{\rm ph}}{p}(1-\frac{m^2_{\rm ph}}{p^2})\right.$$
$$\left.\left.\times\ \ln\frac{1+p/m_{\rm ph}}{1-p/m_{\rm ph}}\right]\right\},$$

$$\tilde{\Sigma}_R(p)=\frac{e^2}{4\pi\mu}\frac{\hat{p}}{p^2}\left(1-\frac{\hat{p}+m_{\rm ph}}{2p}\ln\frac{1+p/m_{\rm ph}}{1-p/m_{\rm ph}}\right). \quad (26)$$

III Comparison with the results in spectral representation

In [3], the one-loop two point functions of CS spinor electrodynamics had been presented in terms of the spectral representation. Regarding the Maxwell term as a higher covariant derivative term, we can consider the results in [3] obtained by Pauli-Villars regularization. If the large topological mass limit is taken, their results should be consistent with ours since both regularization schemes are gauge invariant. The aim of this section is to show it explicitly.

A Polarization tensor

We start from (2.61)–(2.64b) of [3]. After the renormalization, the gauge field propagator is represented in the following spectral form (under the substitutions $\gamma\mu\equiv\tilde{\mu}$, $e^2\gamma\equiv\tilde{e}^2$):

$$\tilde{D}^{(1)}_{\mu\nu}(p)$$
$$=-i\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)\gamma\left[\frac{\tilde{Z}_3}{p^2-\tilde{\mu}^2_{\rm ph}+i\epsilon}+\tilde{\Pi}^{(1)}(p^2)\right]$$
$$+\gamma\tilde{\mu}_{\rm ph}\epsilon_{\mu\nu\alpha}\frac{p^\alpha}{p^2}\left[\frac{\tilde{Z}_3}{p^2-\tilde{\mu}^2_{\rm ph}+i\epsilon}+\tilde{\Pi}^{(2)}(p^2)\right]. \quad (27)$$

The physical mass $\tilde{\mu}_{\rm ph}$ is given by

$$\tilde{\mu}_{\rm ph}=\tilde{\mu}-\frac{\tilde{e}^2\tilde{\mu}}{8\pi}\int_{2m}^{\infty}\frac{1+(4m/a^2)(m-\tilde{\mu})}{a^2-\tilde{\mu}^2}da+O(\tilde{e}^4). \quad (28)$$

The charge renormalization constant $\tilde{Z}_3$ is equal to

$$\tilde{Z}_3=1-\frac{\tilde{e}^2}{8\pi}$$
$$\times\int_{2m}^{\infty}da\,\frac{(1/a^2)(a^2-2m\tilde{\mu})^2+(2m-\tilde{\mu})^2}{(a^2-\tilde{\mu}^2)^2}$$
$$+O(\tilde{e}^4)\,. \quad (29)$$

The continuum contributions are

$$\tilde{\Pi}^{(1)}(p^2)$$
$$=\frac{\tilde{e}^2}{8\pi}\int_{2m}^{\infty}da$$
$$\times\frac{(1/a^2)(a^2-2m\tilde{\mu})^2+(2m-\tilde{\mu})^2}{(p^2-a^2+i\epsilon)(a^2-\tilde{\mu}^2)^2}+O(\tilde{e}^4), \quad (30a)$$

$$\tilde{\Pi}^{(2)}(p^2)$$
$$=\frac{\tilde{e}^2}{4\pi}\left(1-\frac{2m}{\tilde{\mu}}\right)\int_{2m}^{\infty}da$$
$$\times\frac{a^2-2m\tilde{\mu}}{(p^2-a^2+i\epsilon)(a^2-\tilde{\mu}^2)^2}+O(\tilde{e}^4). \quad (30b)$$

The calculation gives:

$$Z_3\equiv\lim_{\gamma\to\infty}\tilde{Z}_3=1,$$
$$\tilde{\mu}_{\rm ph}\equiv\gamma\mu_{\rm ph}=\gamma\mu\left(1+\frac{e^2}{4\pi}\right), \quad (31)$$
$$\Pi^{(1)}(p^2)\equiv\lim_{\gamma\to\infty}\gamma\tilde{\Pi}^{(1)}(p^2)$$
$$=\frac{e^2}{8\pi\mu^2_{\rm ph}}\int_{2m}^{\infty}\frac{da(1+4m^2/a^2)}{a^2-p^2-i\epsilon},$$
$$\Pi^{(2)}(p^2)\equiv\lim_{\gamma\to\infty}\left[-\gamma^2\mu^2\tilde{\Pi}^{(2)}(p^2)\right]$$
$$=\frac{e^2m}{2\pi\mu_{\rm ph}}\int_{2m}^{\infty}\frac{da}{p^2-a^2+i\epsilon}\,. \quad (32)$$

Thus

$$D^{(1)}_{\mu\nu}(p)\equiv\lim_{\gamma\to\infty}\tilde{D}^{(1)}_{\mu\nu}(p)$$
$$=-\frac{1}{\mu_{\rm ph}}\epsilon_{\mu\nu\alpha}\frac{p^\alpha}{p^2}\left[1+\Pi^{(2)}(p^2)\right]$$
$$-i(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2})\Pi^{(1)}(p^2)$$
$$=-i(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2})\Pi^{(1)}(p^2)$$
$$-\frac{1}{\mu_{\rm ph}}\epsilon_{\mu\nu\alpha}\frac{p_\alpha}{p^2}\left[1+\Pi^{(2)}(p^2)-\Pi^{(2)}(0)\right]\,. \quad (33)$$

It is easy to see that the result (32) and (33) for $D^{(1)}_{\mu\nu}$ in the case Pauli-Villars regularization and after taking the limit $\gamma\to\infty$ (modulo a finite renormalization of statistical parameteri μ) coincides with (19), (20) and (21). The crucial feature of (33) is the appearance of the parity-even term $\sim(g_{\mu\nu}-p_\mu p_\nu/p^2)$ in the one-loop approximation. This term has no pole in the complex plane of p^2.

B Electron self-energy

After the substitution $\gamma e^2\equiv\tilde{e}^2$, the spectral form of the fermion propagator will read (see (2.70)–(2.71) of [3]):

$$\tilde{S}^{(1)}(p)=i\left[\frac{\tilde{Z}_2}{\hat{p}-\tilde{m}_{\rm ph}}+\tilde{\Sigma}(p)\right]. \quad (34)$$

The physical mass, $\tilde{m}_{\rm ph}$, is

$$\tilde{m}_{\rm ph}=m+\frac{\tilde{e}^2}{16\pi}\times\int_{-\infty}^{\infty}da\left[\frac{(\tilde{\mu}+2m)(\tilde{\mu}+2a)}{a^2(a-m)}\theta(a^2-M^2)+\frac{(a+m+2\tilde{\mu})(a^2-m^2)}{\tilde{\mu}^2a^2}\times\theta(M^2-a^2)\theta(a^2-m^2)\right]+O(\tilde{e}^4). \quad (35)$$

The fermionic renormalization constant, $\tilde{Z}_2$, is given by

$$\tilde{Z}_2=1-\frac{\tilde{e}^2}{16\pi}\times\int_{-\infty}^{\infty}da\left[\frac{(\tilde{\mu}+2m)(\tilde{\mu}+2a)}{a^2(a-m)^2}\theta(a^2-M^2)+\frac{(a+m+2\tilde{\mu})(a+m)}{\tilde{\mu}^2a^2}\times\theta(M^2-a^2)\theta(a^2-m^2)\right]+O(\tilde{e}^4)\;, \quad (36)$$

where $M=\tilde{\mu}+m$. The continuum contribution in (34) is

$$\tilde{\Sigma}(p)=\frac{\tilde{e}^2}{16\pi}\int_{-\infty}^{\infty}\frac{da}{\hat{p}-a}\times\left[\frac{(\tilde{\mu}+2m)(\tilde{\mu}+2a)}{a^2(a-m)^2}\theta(a^2-M^2)+\frac{(a+m+2\tilde{\mu})(a+m)}{\tilde{\mu}^2a^2}\times\theta(M^2-a^2)\theta(a^2-m^2)\right]+O(\tilde{e}^4). \quad (37)$$

Considering the limit $\gamma\to\infty$ in (34)–(37), we get

$$m_{\rm ph}\equiv\lim_{\gamma\to\infty}\tilde{m}_{\rm ph}=m-\frac{e^2}{2\pi}\left(\gamma+\frac{m}{3\mu}\right),$$
$$Z_2\equiv\lim_{\gamma\to\infty}\tilde{Z}_2=1+\frac{e^2}{4\pi}\frac{5}{3\mu},$$
$$\tilde{\Sigma}_R(p)\equiv\lim_{\gamma\to\infty}\tilde{\Sigma}(p)=\frac{e^2}{4\pi\mu}\frac{\hat{p}}{p^2}\left(1-\frac{\hat{p}+m_{\rm ph}}{2p}\ln\frac{1+p/m_{\rm ph}}{1-p/m_{\rm ph}}\right). \quad (38)$$

One notices that $\tilde{\Sigma}(p)$ in (37) can be written as

$$\tilde{\Sigma}(p)=\tilde{\Sigma}_1(p)+\tilde{\Sigma}_2(p), \quad (39)$$

where

$$\tilde{\Sigma}_1(p)=\frac{\tilde{e}^2}{16\pi}\int_{-\infty}^{\infty}\frac{da}{\hat{p}-a}\left[\left(\frac{\tilde{\mu}^2}{a^2}+\frac{4m}{a}\right)\frac{\theta(a^2-M^2)}{(m-a)^2}+\frac{1}{\tilde{\mu}^2a^2}(a+m)^2\theta(M^2-a^2)\theta(a^2-m^2)\right],$$
$$\tilde{\Sigma}_2(p)=\frac{\tilde{e}^2}{8\pi}\int_{-\infty}^{\infty}\frac{da}{\hat{p}-a}\left[\frac{(a+m)}{(a-m)^2}\frac{\tilde{\mu}}{a^2}\theta(a^2-\hat{M}^2)+\frac{(a+m)}{\tilde{\mu}a^2}\theta(M^2-a^2)\theta(a^2-m^2)\right], \quad (40)$$

where $\tilde{\Sigma}_1(p)$ arises from the exchange of a conventional transverse vector part of the photon, while $\tilde{\Sigma}_2(p)$ comes from the axial part of $\tilde{D}^{(0)}_{\mu\nu}$ in the (5).

It is easily shown that

$$\lim_{\gamma\to\infty}\tilde{\Sigma}_1(p)=0, \quad (41)$$

and thus

$$\lim_{\gamma\to\infty}\tilde{\Sigma}(p)=\lim_{\gamma\to\infty}\tilde{\Sigma}_2(p)=\tilde{\Sigma}_R(p). \quad (42)$$

Therefore, in the limit of the pure CS spinor electrodynamics with Pauli-Villars regularization we get the following result

$$S^{(1)}(p)\equiv\lim_{\gamma\to\infty}\tilde{S}^{(1)}(p)=\frac{iZ_2}{\hat{p}-m_{\rm ph}}+i\tilde{\Sigma}_R(p). \quad (43)$$

Comparing the corresponding results with those in Sect. II, we can see that they are the same.

IV On-shell vertex correction

The one-loop on-shell vertex correction is given by

$$-i\bar{u}(p')\tilde{\Gamma}_\mu(p',p,m)u(p)=\bar{u}(p')\left\{-\gamma e^2\int\frac{d^3q}{(2\pi)^3}\frac{\gamma^\sigma[\hat{p}'+\hat{q}+m]\gamma_\mu[\hat{p}+\hat{q}+m]\gamma^\lambda}{[(p'+q)^2-m^2][(p+q)^2-m^2]}\times\left[\frac{q^2g_{\lambda\sigma}-q_\lambda q_\sigma}{q^2(q^2-\mu^2\gamma^2)}+\frac{i\mu\gamma\epsilon_{\lambda\sigma\rho}q^\rho}{q^2(q^2-\mu^2\gamma^2)}\right]\right\}u(p)\equiv\tilde{J}^a_\mu+\tilde{J}^b_\mu+\tilde{J}^c_\mu, \quad (44)$$

where

$$\tilde{J}^a_\mu=-e^2\gamma\int\frac{d^3q}{(2\pi)^3}\times\frac{[-\hat{q}\gamma_\lambda+2(p'+q)_\lambda]\gamma_\mu[-\gamma^\lambda\hat{q}+2(p+q)^\lambda]}{(q^2-\mu^2\gamma^2)\,[(p'+q)^2-m^2]\,[(p+q)^2-m^2]}, \quad (45)$$

$$\tilde{J}^b_\mu=e^2\gamma\int\frac{d^3q}{(2\pi)^3}\times\frac{[-q^2+2(p'+q)\cdot q]\gamma_\mu[-q^2+2(p+q)\cdot q]}{q^2(q^2-\mu^2\gamma^2)\,[(p'+q)^2-m^2])\,[(p+q)^2-m^2]}, \quad (46)$$

$$\tilde{J}^c_\mu=-e^2\gamma\int\frac{d^3q}{(2\pi)^3}\times\frac{i\mu\gamma\epsilon_{\lambda\sigma\rho}q^\rho[-\hat{q}\gamma^\sigma+2(p'+q)^\sigma]\gamma_\mu[-\gamma^\lambda\hat{q}+2(p+q)^\lambda]}{q^2(q^2-\mu^2\gamma^2)\,[(p'+q)^2-m^2]\,[(p+q)^2-m^2]}. \quad (47)$$

For derivation of (45)–(47), we have used the on-shell condition $\hat{p}=\hat{p'}=m$. and

$$\bar{u}(p')\gamma^\sigma\,[\hat{p}'+\hat{q}+m]=\bar{u}(p')\,[(-\hat{p}'-\hat{q}+m)\gamma^\sigma+2(p'+q)^\sigma]=\bar{u}(p')[-\hat{q}\gamma^\sigma+2(p'+q)^\sigma],$$
$$[\hat{p}+\hat{q}+m]\gamma_\mu u(p)=[-\gamma^\lambda\hat{q}+2(p+q)^\lambda]u(p).$$

The term $\tilde{J}^b_\mu$ is very simple,

$$\tilde{J}^b_\mu = \gamma e^2 \gamma_\mu \int \frac{d^3q}{(2\pi)^3} \frac{1}{(q^2-\mu^2\gamma^2)q^2} = \frac{ie^2}{4\pi\mu}\gamma_\mu \equiv J^b_\mu. \quad (48)$$

The term $\tilde{J}^a_\mu$ can be transformed into the following form

$$\tilde{J}^a_\mu = -\gamma e^2 \int \frac{d^3q}{(2\pi)^3} \quad (49)$$
$$\times \frac{\left[q^2\gamma_\mu - 2\hat{q}q_\mu + 4(p\cdot p' + p\cdot q + p'\cdot q)\gamma_\mu + 4q_\mu m - 4\hat{q}\mathcal{P}_\mu\right]}{(q^2-\mu^2\gamma^2)\left[(p'+q)^2-m^2\right]\left[(p+q)^2-m^2\right]},$$

where $\mathcal{P}_\mu \equiv (p'+p)_\mu$. One can not take the limit $\gamma\to\infty$ directly except the term $4p'\cdot p$, which vanishes after the large-γ limit. However, using the following decomposition

$$\frac{1}{[(k+p)^2-m^2]} = \frac{1}{k^2-m^2} - \frac{2k\cdot p + p^2}{(k^2-m^2)[(k+p)^2-m^2]}, \quad (50)$$

one can see that all the terms in (49) $\sim q$ in the numerator vanish when $\gamma\to\infty$. The first two terms in (49) can be transformed into

$$\tilde{J}^a_\mu = -\gamma e^2 \int \frac{d^3q}{(2\pi)^3} \left\{ \frac{q^2\gamma_\mu - 2\hat{q}q_\mu}{(q^2-\mu^2\gamma^2)(q^2-m^2)^2} \times \left[1 + \frac{(2p'\cdot q + m^2)(2p\cdot q + m^2)}{(2p'\cdot q + q^2)(2p\cdot q + q^2)} - \frac{2p'\cdot q + m^2}{2p\cdot q + q^2} - \frac{2p'\cdot q + m^2}{2p'\cdot q + q^2} \right] \right\}. \quad (51)$$

Only the first term in (51) does not vanish after taking the limit $\gamma\to\infty$. Thus,

$$J^a_\mu \equiv \lim_{\gamma\to\infty} \tilde{J}^a_\mu = \lim_{\gamma\to\infty} \gamma e^2 \int \frac{d^3q}{(2\pi)^3} \frac{2\hat{q}q_\mu - q^2\gamma_\mu}{(q^2-\mu^2\gamma^2)(q^2-m^2)^2}$$
$$= -\lim_{\gamma\to\infty} \frac{\gamma e^2}{3}\gamma_\mu \int \frac{d^3q}{(2\pi)^3} \frac{q^2}{(q^2-\mu^2\gamma^2)(q^2-m^2)^2}$$
$$= -\lim_{\gamma\to\infty} \frac{\gamma e^2}{3}\gamma_\mu \frac{2(\mu\gamma)^3 - 3(\mu\gamma)^2 m + m^3}{8\pi(m^2-\mu^2\gamma^2)^2} = -\frac{ie^2}{4\pi\mu}\frac{1}{3}\gamma_\mu. \quad (52)$$

As for the third term $\tilde{J}^c_\mu$, taking into account that in (47)

$$\epsilon_{\lambda\sigma\rho} q^\rho \hat{q}\gamma^\sigma\gamma_\mu\gamma^\lambda\hat{q} = -2iq_\mu q^2,$$

and after some algebraic manipulation, we have[1]

$$\tilde{J}^c_\mu = 2e^2\mu\gamma^2 \int \frac{d^3q}{(2\pi)^3} \times \frac{[-q_\mu q^2 + (q\cdot p')\gamma_\mu\hat{q} + (q\cdot p)\hat{q}\gamma_\mu + 2m\gamma_\mu q^2 - 2q^2\mathcal{P}_\mu]}{q^2(q^2-\tilde{\mu}^2)[(p'+q)^2-m^2][(p+q)^2-m^2]}$$
$$= 2e^2\mu\gamma^2 \int \frac{d^3q}{(2\pi)^3} \times \frac{-2q_\mu q^2 + (2m\gamma_\mu - 2\mathcal{P}_\mu)q^2 + \left[\hat{q}\gamma_\mu(2p\cdot q + q^2)/2 + \gamma_\mu\hat{q}(2p'\cdot q + q^2)/2\right]}{q^2(q^2-\mu^2\gamma^2)[(p'+q)^2-m^2][(p+q)^2-m^2]}$$
$$= 2e^2\mu\gamma^2 \int \frac{d^3q}{(2\pi)^3} \left\{ \frac{2m\gamma_\mu - 2q_\mu - 2\mathcal{P}_\mu}{(q^2-\mu^2\gamma^2)(2p'q+q^2)(2pq+q^2)} + \frac{\gamma_\mu\hat{q}}{2q^2(q^2-\mu^2\gamma^2)[(p+q)^2-m^2]} + \frac{\hat{q}\gamma_\mu}{2q^2(q^2-\mu^2\gamma^2)[(p'+q)^2-m^2]} \right\}. \quad (53)$$

Similar to (49), for the terms $\sim q$ in (53) one cannot take the large-γ limit directly, we still need first to employ the manipulation (50). Considering the symmetry of integrand, we get

$$J^c_\mu \equiv \lim_{\gamma\to\infty} \tilde{J}^c_\mu = -\frac{e^2}{\mu} \int \frac{d^3q}{(2\pi)^3} \left[\frac{4(m\gamma_\mu - \mathcal{P}_\mu - k_\mu)}{(2p\cdot q + q^2)(2p'\cdot q + q^2)} + \frac{\gamma_\mu\hat{q}}{q^2(2p\cdot q + q^2)} + \frac{\gamma_\mu\hat{q}}{q^2(2p'\cdot q + q^2)} \right]. \quad (54)$$

The standard Feynman integration gives that

$$J^c_\mu = -2\frac{e^2}{\mu} \int \frac{d^3q}{(2\pi)^3} \int_0^1 dx \left\{ \frac{(2m\gamma_\mu - 2\mathcal{P}_\mu) - 2(q_\mu - x\mathcal{P}_\mu)}{[q^2 - m^2 + x(1-x)K^2]^2} + \frac{\gamma_\mu(\hat{q} - x\hat{p})}{(q^2 - m^2x^2)^2} \right\}$$
$$= -\frac{2e^2 i}{\mu 8\pi} \int_0^1 dx \left\{ \frac{2m\gamma_\mu - 2(1-x)\mathcal{P}_\mu}{[m^2 - x(1-x)K^2]^{1/2}} - \gamma_\mu \right\}$$
$$= \frac{ie^2}{4\pi\mu} \left[\gamma_\mu - \frac{2m\gamma_\mu - \mathcal{P}_\mu}{K} \ln\frac{1 + K/(2m)}{1 - K/(2m)} \right]$$
$$= \frac{ie^2}{4\pi\mu} \left[\gamma_\mu - \frac{i\epsilon_{\mu\nu\lambda}K^\nu\gamma^\lambda}{K} \ln\frac{1 + K/(2m)}{1 - K/(2m)} \right], \quad (55)$$

where $K_\mu \equiv p'_\mu - p_\mu$, $K \equiv \sqrt{K^2}$ and we have used the three-dimensional analogue of the Gordon identity:

$$\gamma_\mu = \frac{1}{2m}\left[\mathcal{P}_\mu + i\epsilon_{\mu\nu\lambda}K^\nu\gamma^\lambda\right]. \quad (56)$$

Thus at $\gamma\to\infty$, from the (48), (52) and (55) we get

$$\lim_{\gamma\to\infty}\left(-i\tilde{\Gamma}_\mu(K)\right) \equiv -i\Gamma_\mu(K) = J^a_\mu + J^b_\mu + J^c_\mu,$$
$$\Gamma_\mu(K) = -\frac{e^2}{4\pi\mu}\left[\frac{5}{3}\gamma_\mu - \frac{i\epsilon_{\mu\nu\lambda}K^\nu\gamma^\lambda}{K} \ln\frac{1 + K/(2m)}{1 - K/(2m)} \right]$$
$$= \gamma_\mu F_1(K^2) + i\epsilon_{\mu\nu\lambda}K^\nu\gamma^\lambda F_2(K^2). \quad (57)$$

The vertex renormalization is defined as

$$\Gamma_\mu(K) = \gamma_\mu(Z_1^{-1} - 1) + Z_1^{-1}\Gamma^R_\mu(K) \quad (58)$$

[1] In the numerator of (53) we skip the term $\sim \epsilon_{\lambda\sigma\rho}q^\rho p'^\sigma p^\lambda$ coming from (47), since after integration it will become of the form $\epsilon_{\lambda\sigma\rho}p^\rho p^\lambda p'^\sigma$ and $\epsilon_{\lambda\sigma\rho}p'^\rho p^\lambda p'^\sigma$, both giving zero

and the renormalization condition is as usual

$$\Gamma_\mu^R(K)|_{\hat{p}=\hat{p}'=m,\ K_\alpha=p'_\alpha-p_\alpha=0} = 0. \tag{59}$$

Then we get the vertex renormalization constant

$$\begin{aligned} Z_1^{-1}\gamma_\mu &= \gamma_\mu + \gamma_\mu F_1(0), \\ {Z_1}^{-1} &= 1 - \frac{e^2}{4\pi\mu}\frac{5}{3}, \end{aligned} \tag{60}$$

and the one-loop radiative correction to the vertex as

$$\begin{aligned} \Gamma_\mu^R(K) &= -\gamma_\mu + Z_1(\gamma_\mu + \Gamma_\mu) \\ &= \frac{ie^2}{4\pi\mu}\epsilon_{\mu\nu\lambda}K^\nu\gamma^\lambda\frac{1}{K}\ln\frac{1+K/(2m)}{1-K/(2m)}. \end{aligned} \tag{61}$$

From (26) we have

$$Z_1 = 1 + \frac{e^2}{4\pi\mu}\frac{5}{3} = Z_2, \tag{62}$$

which is just the consequence of Ward identity

$$K^\mu\Gamma_\mu(K) = \Sigma(p') - \Sigma(p). \tag{63}$$

It is remarkable that $\Gamma_\mu^R(K^2=0)$ does not vanish, i.e.

$$\Gamma_\mu^R(0) = \frac{ie^2}{4\pi}\frac{1}{\mu m}\epsilon_{\mu\nu\lambda}K^\nu\gamma^\lambda = i\frac{\alpha}{\mu m}\epsilon_{\mu\nu\lambda}K^\nu\gamma^\lambda, \tag{64}$$

which gives the three-dimensional analogue of Schwinger's result for the anomalous magnetic moment of the electron. In a slowly varying (in both space and time) external electricmagnetic field, it will lead to a new interaction Hamiltonian[2]:

$$\begin{aligned} \Delta\mathcal{H} &= -\frac{\alpha}{m\mu}\epsilon^{\mu\nu\lambda}\bar{\psi}(x)\gamma_\lambda\psi(x)\partial_\nu A_\mu \\ &= -\frac{\alpha}{2m\mu}\epsilon^{\mu\nu\lambda}\bar{\psi}(x)\gamma_\lambda\psi(x)F_{\mu\nu} \\ &= -\frac{\alpha}{2m\mu}\bar{\psi}(x)\sigma^{\mu\nu}\psi(x)F_{\mu\nu}, \end{aligned} \tag{65}$$

where we have used that

$$\epsilon_{\mu\nu\lambda}\gamma^\lambda = \frac{i}{2}[\gamma_\mu, \gamma_\nu] \equiv \sigma_{\mu\nu}. \tag{66}$$

Thus this term leads to the anomalous magnetic moment of the electron [14], which is consistent with the result in [11]. It is very interesting that this term exists in scalar case too [15].

V Pure Chern-Simons electrodynamics

Now we consider the case of pure CS electrodynamics, i.e. put $\gamma\to\infty$ at the tree level. The vacuum polarization tensor and $D_{\mu\nu}^{(1)}(p)$ will be the same since this does not change the electron loop. However, the electron self-energy and the vertex correction will be different since the gauge field propagator is replaced by (6).

We first consider the electron self-energy

$$\begin{aligned} -i\Sigma^{\text{pure}}(p) &= \frac{ie^2}{\mu}\int\frac{d^nq}{(2\pi)^n}\frac{\gamma_\nu(\hat{q}+\hat{p}+m)\gamma_\mu\epsilon^{\mu\nu\rho}q_\rho}{q^2[(q+p)^2-m^2]} \\ &= -\frac{2e^2}{\mu}\int\frac{d^nq}{(2\pi)^n}\frac{q^2+(\hat{p}-m)\hat{q}}{q^2[(q+p)^2-m^2]} \\ &= \frac{ie^2}{4\pi\mu}\left\{2m-(\hat{p}-m)\frac{\hat{p}}{m}\left[\frac{m^2}{p^2}+\frac{m^3}{2p^3}\right.\right. \\ &\quad \times\left.\left.\left(1-\frac{p^2}{m^2}\right)\ln\frac{1-p/m}{1+p/m}\right]\right\}. \end{aligned} \tag{67}$$

Similar discussions as the ones used in getting (26) give that

$$\begin{aligned} m_{\text{ph}}^{\text{pure}} &= m(1+\frac{e^2}{2\pi}\frac{1}{\mu}), \\ Z_2^{\text{pure}} &= 1+\frac{e^2}{4\pi}\frac{1}{\mu}, \\ \Sigma_R^{\text{pure}}(p) &= -\frac{e^2}{4\pi\mu}(\hat{p}-m_{\text{ph}})\left\{\frac{\hat{p}}{m_{\text{ph}}}\left[\frac{m_{\text{ph}}^2}{p^2}+\frac{m_{\text{ph}}^3}{2p^3}\right.\right. \\ &\quad \times\left.\left.\left(1-\frac{p^2}{m_{\text{ph}}^2}\right)\ln\frac{1-p/m_{\text{ph}}}{1+p/m_{\text{ph}}}\right]-1\right\}, \\ \tilde{\Sigma}_R^{\text{pure}}(p) &= \frac{e^2}{4\pi}\frac{1}{\mu}\frac{\hat{p}}{p^2}\left[1+\frac{\hat{p}+m_{\text{ph}}}{2p^3}\ln\frac{1-p/m_{\text{ph}}}{1-p/m_{\text{ph}}}\right]. \end{aligned} \tag{68}$$

Using the techniques stated above, the on-shell vertex correction is given as follows

$$\begin{aligned} &-i\bar{u}(p')\Gamma_\mu^{\text{pure}}(p',p,m)u(p) \equiv -i\Gamma_\mu^{\text{pure}}(K) \\ &= \frac{ie^2}{\mu}\int\frac{d^nq}{(2\pi)^n}\frac{\gamma_\rho(\hat{q}+\hat{p}'+m)\gamma_\mu(\hat{q}+\hat{p}+m)\gamma_\nu\epsilon^{\nu\rho\lambda}q_\lambda}{q^2[(q+p')^2-m^2][(q+p)^2-m^2]} \\ &= -\frac{2e^2}{\mu}\int\frac{d^nq}{(2\pi)^n}\frac{q{\cdot}p'\gamma_\mu\hat{q}+q{\cdot}p\hat{q}\gamma_\mu+2q^2[m\gamma_\mu-2\mathcal{P}_\mu-q_\mu]}{q^2[(q+p)^2-m^2][(q+p')^2-m^2]} \\ &= -\frac{2e^2}{\mu}\int\frac{d^nq}{(2\pi)^n}\left[\frac{\gamma_\mu\hat{q}}{2q^2(q^2+2p{\cdot}q)}+\frac{\hat{q}\gamma_\mu}{2q^2(q^2+2p'{\cdot}q)}\right. \\ &\quad\left.+\frac{2m\gamma_\mu-2\mathcal{P}_\mu-2q_\mu}{(q^2+2p'{\cdot}q)(q^2+2p{\cdot}q)}\right] \\ &= \frac{ie^2}{4\pi\mu}\left[\gamma_\mu-(2m\gamma_\mu-\mathcal{P}_\mu)\frac{1}{K}\ln\frac{1+K/(2m)}{1-K/(2m)}\right] \\ &= \frac{ie^2}{4\pi\mu}\left[\gamma_\mu-i\epsilon_{\mu\nu\lambda}K^\nu\gamma^\lambda\frac{1}{K}\ln\frac{1+K/(2m)}{1-K/(2m)}\right], \end{aligned} \tag{69}$$

where the three-dimensional Gordon identity (56) has been used. Correspondingly, the vertex renormalization constant is

$$Z_1^{\text{pure}} = 1+\frac{e^2}{4\pi}\gamma_\mu, \tag{70}$$

and we still have

$$Z_1^{\text{pure}} = Z_2^{\text{pure}}. \tag{71}$$

[2] The self-energy insertion in the external line can be disregarded since the electrons are on mass-shell

In particular, we still obtain the same anomalous magnetic moment term.

VI Conclusion and discussion

We have made a detailed study of the quantum correction to CS spinor electrodynamics. We give complete analytical results for one-loop quantum corrections such as polarization tensor, electron self-energy and specially for the on-shell vertex. We find the three dimensional analogue of the Schwinger anomalous magnetic term, despite it is in the second order, this may lead to nontrivial planar dynamics since it can provide new interaction between charged particles. We compare the different procedure of taking the limit $\gamma \to \infty$ and verify explicitly that in both cases the Ward identity is satisfied, and hence that the physical quantities are independent of the order of taking large-γ limit.

In addition, in both cases, the results are finite and the β-function vanishes identically. If we take into account the higher order perturbative corrections, according to BPHZ renormalization procedure, we believe that the results are still finite since the one-loop renormalization constants are all finite and all propagators and vertex part in the asymptotic region will be the same as those in the free case after renormalization.

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